

MATH 2050C Mathematical Analysis I
2020-21 Term 2
Problem Set 6

due on Mar 4, 2022 (Friday) at 11:59PM

Instructions: You are allowed to discuss with your classmates or seek help from the TAs but you are required to write/type up your own solutions. You can either type up your assignment or scan a copy of your written assignment into ONE PDF file and submit through CUHK Blackboard on/before the due date. Please remember to write down your name and student ID. You can refer to the webpage under "Useful Links" below about how to submit assignments through Blackboard. **No late homework will be accepted.** All the exercises below are taken from the textbook.

Required Readings: Chapter 3.3

Optional Readings: none

Problems to hand in

Section 3.3: Exercise # 4, 7, 10, 12(b)

Suggested Exercises

Section 3.3: Exercise # 1, 2, 3, 5, 6, 8, 9, 11, 12(a)(c)(d)

Challenging Exercises (optional)

1. Let $x_1 := \sqrt{2}$ and for $n \geq 2$,

$$x_n := \sqrt{2 + \sqrt{x_{n-1}}}.$$

Prove that (x_n) is convergent.

2. Let $a > \sqrt{2}$ be a fixed number. Define $x_1 := a$, and for $n \geq 2$,

$$x_n := \frac{2 + x_{n-1}}{1 + x_{n-1}}.$$

- (a) Show that $x_1 > x_3 > x_5 > \dots$.
- (b) Show that $x_2 < x_4 < x_6 > \dots$.
- (c) Prove that $\lim(x_n) = \sqrt{2}$.